# Indicators for K-12 Mathematics: Algebra 2

***Algebra 2*** continues students' study of advanced algebraic concepts including functions, polynomials, rational expressions, systems of functions and inequalities, and matrices. Students will be expected to describe and translate among graphic, algebraic, numeric, and verbal representations of relations and use those representations to solve problems. Emphasis should be placed on practical applications and modeling. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Prerequisites

- ***Operate with matrices to solve problems.***
- ***Create linear models, for sets of data, to solve problems.***
- ***Use linear functions and inequalities to model and solve problems.***
- ***Use quadratic functions to model problems and solve by factoring and graphing.***
- ***Use systems of linear equations or inequalities to model and solve problems.***
- ***Graph and evaluate exponential functions to solve problems.***

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# Algebra 2

**GOAL 1: The learner will perform operations with complex numbers, matrices, and polynomials.**

1.01 Simplify and perform operations with rational exponents and logarithms (common and natural) to solve problems.
1.02 Define and compute with complex numbers.
1.03 Operate with algebraic expressions (polynomial, rational, complex fractions) to solve problems.
1.04 Operate with matrices to model and solve problems.
1.05 Model and solve problems using direct, inverse, combined and joint variation.

**GOAL 2: The learner will use relations and functions to solve problems.**

2.01 Use the composition and inverse of functions to model and solve problems; justify results.
2.02 Use quadratic functions and inequalities to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.
2.03 Use exponential functions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.
2.04 Create and use best-fit mathematical models of linear, exponential, and quadratic functions to solve problems involving sets of data.
a) Interpret the constants, coefficients, and bases in the context of the data.
b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.
2.05 Use rational equations to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.
c) Identify the asymptotes and intercepts graphically and algebraically.
2.06 Use cubic equations to model and solve problems.
a) Solve using tables and graphs.
b) Interpret constants and coefficients in the context of the problem.
2.07 Use equations with radical expressions to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the degree, constants, and coefficients in the context of the problem.
2.08 Use equations and inequalities with absolute value to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.
2.09 Use the equations of parabolas and circles to model and solve problems; justify results.
a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.
2.10 Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.

***Vocabulary Concepts Skills***

Inverse Relationship

Laws of Exponents

Laws of Logarithms

Base

Common Logarithms

Natural Logarithms

***e***

Radical

# 1.01 Simplify and perform operations with rational exponents and logarithms (common and natural) to solve problems.

A. Given that log 2.72 = 0.4346, approximate the following without a calculator: log 0.272, log 272, and log 0.00272.

B. Simplify: $\frac{1+5^{\frac{1}{2}}}{3-5^{\frac{1}{2}}}$ and $\frac{1+\sqrt{5}}{3-\sqrt{5}}$.

C. Rewrite $4 = 3^x$ in logarithmic forms (base 3, base 10, and base *e*).

D. Solve $663 = 49(\ 2.165^x)$ for $x$. Justify each step.

E. Solve $350 = 200e^{2r}$ for $r$. Justify each step.

F. Solve $e^{\frac{1}{2}x}e^{3x} = 5$ for x. Justify each step.

G. The number of airline passengers increased from 465.6 million in 1990 to 614.3 million in 1998. What was the average annual growth rate (percent) for the 1990-98 period?

H. The wind chill (WC) is a measure of the heat loss from the body when temperature and wind speed are combined. The relationship is expressed algebraically as follows:
$WC\ (^{o}F) = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275T(V^{0.16})$ where
T = air temperature ($^{o}$F) and V = wind speed (mph).
A wind chill less than -20$^{o}$F is dangerous because human flesh can freeze within one minute. What wind speed will generate a wind chill of -20$^{o}$F when the air temperature is 6$^{o}$F? What air temperature will generate a wind chill of 32$^{o}$F when the wind speed is 35 mph?

I. The median family income in 1947 was $3031. According to the US Census Bureau, that same income in 1997 dollars is $20,102. Determine the annual inflation rate (percent) for the period 1947-1997.

***Vocabulary Concepts Skills***

Imaginary Number

Addition

Subtraction

Multiplication

a + b$i$

Conjugate

# 1.02 Define and compute with complex numbers.

A. Simplify: $(3 + 2i)^2$

B. Simplify: $i^4$

C. Simplify: $5(2 + 4i) - 2(5 - 8i)$

D. Simplify: $\frac{2+4i}{5-i}$

E. Identify the conjugate of $-2 + i$.

F. Solve for **a** and **b** if $(\mathbf{a} + \mathbf{b}i)^2 = 5 + 12i$.

G. Simplify: $\frac{3+4i}{2-i} + \frac{6-5i}{2+2i}$

H. $i^{797} + i^{293} = ?$

***Vocabulary Concepts Skills***

Polynomial

Rational Expression

Complex Fractions

Simplify

Addition

Subtraction

Multiplication

Synthetic Division

Division Algorithm

Factor

Binomial Theorem

# 1.03 Operate with algebraic expressions (polynomial, rational, complex fractions) to solve problems.

A. Expand and simplify $(3x - 2y)^4$

B. Find the quotient: $(x^3 - 3x^2 - 4x + 15) \div (x + 5)$

C. Find the quotient: $(4x^3 - 7x + 5) \div (2x - 1)$

D. Factor completely: $4x^2 - 16y^2$

E. Factor completely: $27x^3 + 8$

F. Factor completely: $6x^2 + 13x - 5$

G. Factor completely: $x^4 - 1$

H. Factor completely: $3xy + 6y + 4x + 8$

I. Simplify: $\dfrac{x^2 - x - 56}{x^2 + 10x + 21}$

J. Simplify: $\dfrac{9 - a^2}{a^2 + a - 12}$

K. Simplify: $\dfrac{x^2 + 7x + 12}{x^2 - 9} \cdot \dfrac{x - 3}{x + 3}$

L. Simplify: $\dfrac{x^2 - y^2}{12} \div \dfrac{y^4 - x^4}{3y^2 + 3x^2}$

M. Simplify: $\dfrac{4x + 9}{4x - 5} + \dfrac{x - 3}{x + 1} - \dfrac{34 - 2x}{4x^2 - x - 5}$

N. Simplify: $\dfrac{5 + \dfrac{2}{x - 1}}{2 - \dfrac{5}{x - 1}}$

*Vocabulary*
*Concepts*
*Skills*

Ordered Array

Dimensions (rows, columns)

Element

Identity Matrix

Inverse Matrix

Matrix Addition

Matrix Subtraction

Matrix Multiplication

Scalar Multiplication

Determinant

# 1.04 Operate with matrices to model and solve problems.

A. (1, 7), (6, -2), (11, 3), and (15, -6) are points on the graph of $y = ax^3 + bx^2 + cx + d$. Set up the matrix equation to determine a, b, c, and d. What is the equation? What is $A^{-1}$?

B. Without a calculator find $\begin{vmatrix} 3 & 7 \\ -4 & 0.5 \end{vmatrix}$.

C. Given $A = \begin{bmatrix} 0.5 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} a & 6 & -2 & 8 \\ 4 & -3 & b & 8 \\ 7 & 0 & 1 & c \end{bmatrix}$, and

$AB = \begin{bmatrix} -15 & -6 & 22 & 44 \end{bmatrix}$, find a, b, and c.

D. $T = \begin{bmatrix} 560.0 & 72.8 & 1327.1 & 349.3 \\ 582.8 & 71.8 & 1384.2 & 370.0 \end{bmatrix}$ and $P = \begin{bmatrix} 41131.96 \\ 42484.09 \\ 31218.12 \\ 17720.14 \end{bmatrix}$.

Matrix T shows the number of employees (in thousands) in several areas of the transportation industry (by columns: air, water, truck, and ground passenger) for 1998 and 1999 (rows). Matrix P shows the average annual salary for employees in each transportation area (by rows: air, water, truck, and ground passenger). Find TP and define the elements.

***Vocabulary Concepts Skills***

Independent

Dependent

# 1.05 Model and solve problems using direct, inverse, combined and joint variation.

A. The formula for the volume (*V*) of a frustum of a cone (a horizontal slice of a cone) is given by $V = \frac{1}{3}h(B_1 + B_2 + \sqrt{B_1 B_2})$, where *h* is the height, $B_1$ is the area of the lower base, and $B_2$ is the area of the lower base. Describe the volume as a variation with respect to the independent variables. Rewrite the formula for height in terms of the volume and areas of the bases and describe the height as a variation in terms of those quantities.

B. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If the resistance of 175 meters of wire having a diameter of 0.4 centimeters is 1.5 ohms, find the resistance of 300 meters of wire having a diameter of 0.25 centimeters.

C. The period of a simple pendulum varies directly as the square root of its length. If a pendulum three feet long has a period of 4.8 seconds, find the period of a pendulum half as long.

D. The Hubble Telescope could see stars and galaxies whose brightness is approximately 2% of the faintest objects observable using Earth-based telescopes. The brightness of an object varies inversely as the square of its distance from the observer. How much farther into space is the Hubble Telescope able to see compared to the Earth-based telescopes?

***Vocabulary***
***Concepts***
***Skills***

$f^{-1}(x)$

$(f \circ g)(x) = f(g(x))$

Identity
Function
$y = x$

# 2.01 Use the composition and inverse of functions to model and solve problems; justify results.

A. Given: $f(x) = 5x - 3$ and $g(x) = 6 - x^2$. Find each of the following:

$$f(g(x)),\ g(f(x)),\ (f \circ g)(-2),\ (g \circ f)(3),\ f^{-1}(x),\ g^{-1}(f(x))$$

B. Find $f(g(x))$ and $(g \circ f)(x)$ for $f(x) = 3x - 2$; $g(x) = x - 1$

C. Find $f(g(x))$ and $(g \circ f)(x)$ for $f(x) = x^2 - 1$; $g(x) = \dfrac{1}{x-1}$

D. Find $f(g(x))$ and $(g \circ f)(x)$ for $f(x) = x^2 - 2$; $g(x) = \sqrt{x+1}$

E. Find $f^{-1}(x)$ for $f(x) = 2x - 4$. Generate a table of values for $f^{-1}(x)$ and $f(x)$ to confirm the inverse relationship. Explain how the inverse relationship can be confirmed graphically.

F. Algebraically confirm that $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$ are inverse functions.

G. The following is a table of corresponding Fahrenheit/Celsius temperatures collected from a cooking thermometer.

| °F | °C | °F | °C | °F | °C | °F | °C |
|---|---|---|---|---|---|---|---|
| 400 | 204 | 320 | 160 | 245 | 118 | 180 | 82 |
| 385 | 196 | 300 | 148 | 230 | 110 | 170 | 76 |
| 370 | 188 | 285 | 140 | 220 | 104 | 155 | 68 |
| 350 | 176 | 270 | 132 | 210 | 98 | 140 | 60 |
| 335 | 168 | 260 | 126 | 195 | 90 | 130 | 54 |

Using a calculator, determine the best-fit linear equation that models the relationship between Fahrenheit ($x$) and Celsius ($y$). Determine a second best-fit linear equation that models the relationship between Celsius ($x$) and Fahrenheit ($y$). How are the slopes of the two linear equations related? Locate the intersection of the two equations. What does this point represent in the context of the data? Graph the indentity equation $y = x$. What is the relationship between the two best-fit equations with respect to the identity equation? Confirm algebraically that the best-fit equations are inverses of one another.

***Vocabulary***
***Concepts***
***Skills***

Graph

Factor

Completing the Square

Quadratic Formula

Set Notation

Number Line

Maximum

Minimum

Increasing

Decreasing

Domain

Range

Independent

Dependent

$f(x) = ax^2 + bx + c$

$y - k = a(x - h)^2$

# 2.02 Use quadratic functions and inequalities to model and solve problems; justify results.

A. Name the vertex and all intercepts of $f(x) = x^2 - 4x + 3$. Justify the results algebraically.

B. Name the vertex and all intercepts of $f(x) = 3x^2 - 4x + 5$. Justify the results algebraically.

C. Determine the solution set for $x^2 > 2(x + 4)$. Justify the results algebraically.

D. For $f(x) = 2x^2 + \mathbf{b}x + 10$: As **b** increases/decreases, how does the graph of $f(x)$ change?

E. For $f(x) = \mathbf{a}x^2 - 2x + 5$: As **a** gets close to zero, how does the graph of $f(x)$ change?

F. Compare $f(x) = \mathbf{a}x^2 + bx + c$ and $g(x) = \mathbf{-a}x^2 + bx + c$. Identify similarities and differences.

G. For $f(x) = 2.5x^2 + 17x + \mathbf{c}$: As **c** increases/decreases, how does the graph of $f(x)$ change?

H. What is a quadratic function that has roots $(5 + 2i)$ and $(5 - 2i)$?

I. The function $f(x) = -0.019x^2 + 3.04x - 58.87$ describes newspaper circulation (millions) in the United States for 1920-98 ($x = 20$ for 1920). Identify periods of increasing and decreasing circulation. According to the function, when did newspaper circulation peak? When will circulation reach 45 million?

J. From the top of a 56 foot tower, a projectile is launched straight up and reaches a maximum height of 120 feet after two seconds. What is the equation of the height function $h(t)$ in terms of time $t$?
Use the form $h(t) = a(t - b)^2 + c$. Explain the coefficient **a**. When will the projectile hit the ground? During what interval was the projectile at a height of at least 96 feet?

***Vocabulary Concepts Skills***

(continued)

Parabola

Transformations

Coefficients

Vertex

Intercepts

Solutions

Roots

Zeros

Properties of Equality

K. The space shuttle uses solid rocket boosters (SRB) during the launch phase of its flight from Cape Canaveral. The SRBs burn for about two minutes, shut down, detach from the main rocket assembly, and fall back to Earth 140 miles downrange. Parachutes assist the ocean landing, beginning at an altitude of 20,000 feet. The SRBs are recovered and used again for a later launch. The function $f(x) = -33 + 0.72x - 0.00176x^2$, for $x \geq 115.5$, describes the altitude of the shuttle's SRBs since the launch ($x$ is elapsed time in seconds). How long after launch do the SRBs splash down? The SRBs separate from the shuttle after 115.5 seconds. How much longer do the SRBs continue to gain altitude?

L. For $f(x) = 2x^2 - 17x + 26$, locate the vertex and intercepts. Justify the results algebraically.

M. Solve $3x^2 - 5 = 7x$. Justify the results algebraically.

N. Solve $x^2 + 3x \leq 28$. Justify the results algebraically.

O. The height of a baseball, just hit, is approximated by $h(x) = -0.0011x^2 + 0.4545x + 3$ where $x$ is the horizontal distance from homeplate. The ball is approaching the ten-foot outfield wall, 400 feet from homeplate. If the ball is not caught, will it go over the wall?

P. Each orange tree in a California grove produces 600 oranges per year if no more than 20 trees are planted per acre. For each additional tree planted per acre, the yield per tree decreases by 15 oranges. How many trees per acre should be planted to obtain the greatest number of oranges?

*Vocabulary*
*Concepts*
*Skills*

Graph

Increasing

Decresasing

Domain

Range

Independent

Dependent

Laws of Exponents

Laws of Logarithms

Inverse Relationship

Coefficient

Base

***e***

Initial Value

Intercepts

Common Logarithm

Natural Logarithm

Properties of Equality

$f(x) = a \cdot b^x + c$

# 2.03 Use exponential functions to model and solve problems; justify results.

A. Bacteria growing on disgarded food triples every five hours. If there are one million bacteria present now, how many will there be one day later?

B. At the end of four years ($t$), a savings account paying 5.35% annually ($r$) compounded continuously, had a balance ($B$) of $3096.56. What was the initial deposit ($P$)? (Use $B = Pe^{rt}$) If the initial deposit had been in an account compounded annually, how much less interest would have been earned?

C. Over the last year the stock value of an internet company has dropped at a rate of 17% per month. The value of the stock at the beginning of the year was $19.50. What was the value of the stock at the end of the year? If the stock's value continues to decrease at the same rate, how long does it take the stock to be worth one-tenth of its original value?

D. In 1998 there were 429,000 people employed in the United States as computer support specialists. By 2002 that number grew to 507,000. Assuming a constant annual growth rate in the number of specialists, how many will there be in 2006? When will the number of computer support specialists exceed one million?

E. If $f(x) = 5 \cdot \mathbf{b}^x$ and $\mathbf{b} > 1$, as **b** increases, how does the graph of $f(x)$ change?

F. If $f(x) = 5 \cdot \mathbf{b}^x$ and $0 < \mathbf{b} < 1$, as **b** approaches zero, how does the graph of $f(x)$ change?

G. If $f(x) = \mathbf{a} \cdot 1.9^x$ and $\mathbf{a} \geq 1$, as **a** increases, how does the graph of $f(x)$ change?

H. If $f(x) = 2 \cdot 3.1^x - 10$ locate exactly the $x$- and $y$-intercepts.

***Vocabulary Concepts Skills***

Scatter Plot

Residuals

Independent

Dependent

Domain

Range

Scatter plot

Regression

Correlation Coefficient

$R^2$

Estimation

Prediction

Interpolation

Extrapolation

# 2.04 Create and use best-fit mathematical models of linear, exponential, and quadratic functions to solve problems involving sets of data.

A. For any 4-sided convex polygon, two distinct diagonals can be drawn. For any 5-sided convex polygon, five distinct diagonals can be drawn. For any 6-sided convex polygon, nine distinct diagonals can be drawn. How many distinct diagonals can be drawn in a 20-sided polygon? Create the function that will generate the number of distinct diagonals for $n$-sided polygons.

B. The winnings for the 2003 *Coca-Cola 600* are shown. Create an algebraic model of the data. Describe the average change in winnings and how it relates to the data. What are other variables which affect the prizes at an automobile race or any other professional athletic event?

| Place | Winnings | Place | Winnings |
|---|---|---|---|
| 1 | 271,900 | 9 | 129,153 |
| 2 | 206,500 | 10 | 120,331 |
| 3 | 184,633 | 11 | 84,740 |
| 4 | 137,350 | 12 | 117,542 |
| 5 | 186,850 | 13 | 119,003 |
| 6 | 112,875 | 14 | 104,600 |
| 7 | 132,225 | 15 | 110,950 |
| 8 | 135,103 | 16 | 76,500 |

C. Graph and describe the newspaper circulation (in millions) data shown. What variables affect newspaper circulation? Create an algebraic model of the data (let $x = 20$ for 1920). According to the model, will the newspaper circulation drop below 50 million? If it does, when?

| | | | | | |
|---|---|---|---|---|---|
| 1920 | 27.8 | 1960 | 58.9 | 1992 | 60.2 |
| 1925 | 33.7 | 1965 | 60.4 | 1993 | 59.8 |
| 1930 | 39.6 | 1970 | 62.1 | 1994 | 59.3 |
| 1935 | 38.2 | 1975 | 60.7 | 1995 | 58.2 |
| 1940 | 41.1 | 1980 | 62.2 | 1996 | 57.0 |
| 1945 | 48.4 | 1985 | 62.8 | 1997 | 56.7 |
| 1950 | 53.8 | 1990 | 62.3 | 1998 | 56.2 |
| 1955 | 56.1 | 1991 | 60.7 | | |

***Vocabulary Concepts Skills***

D. Since the 1924 Olympics, men and women have competed separately in 400 Meter Free Style swimming events. Compare the data (all results in seconds) for the two events and describe similarities and differences. Determine best-fit models for each of the men's and women's data. Which group of athletes is making the greater improvement? Explain. Assuming winning performances occur according to the models, what will be the results for the 2004 Athens Olympics? According to the models, will the women's performance ever equal or exceed the men's performance in this event? If so, when? If not, why? Include other data to support your hypothesis.
Does the number of total participants correlate well with athletic performance? Explain. Identify and discuss variables that affect athletic performance.

| Performance | Men | Women |
|---|---|---|
| 1924 | 304.2 | 362.2 |
| 1928 | 301.6 | 342.8 |
| 1932 | 288.4 | 328.5 |
| 1936 | 284.5 | 326.4 |
| 1948 | 281.0 | 317.8 |
| 1952 | 270.7 | 312.1 |
| 1956 | 267.3 | 294.6 |
| 1960 | 258.3 | 290.6 |
| 1964 | 252.2 | 283.3 |
| 1968 | 249.0 | 271.8 |
| 1972 | 240.27 | 259.04 |
| 1976 | 231.93 | 249.89 |
| 1980 | 231.31 | 248.76 |
| 1984 | 231.23 | 247.10 |
| 1988 | 226.95 | 243.85 |
| 1992 | 225.00 | 247.18 |
| 1996 | 227.95 | 247.25 |
| 2000 | 220.59 | 245.80 |

| Total Participation | Men | Women |
|---|---|---|
| 1924 | 2956 | 136 |
| 1928 | 2724 | 290 |
| 1932 | 1281 | 127 |
| 1936 | 3738 | 328 |
| 1948 | 3714 | 385 |
| 1952 | 4407 | 518 |
| 1956 | 3003 | 397 |
| 1960 | 4738 | 610 |
| 1964 | 4457 | 683 |
| 1968 | 4750 | 781 |
| 1972 | 6659 | 1171 |
| 1976 | 4915 | 1274 |
| 1980 | 4320 | 1192 |
| 1984 | 5458 | 1620 |
| 1988 | 6983 | 2438 |
| 1992 | 7555 | 3008 |
| 1996 | 7060 | 3684 |
| 2000 | 6582 | 4069 |

***Vocabulary Concepts Skills***

Graph

Increasing

Decreasing

Domain

Range

Asymptotes

Discontinuity

Factor

Common Denominator

Properties of Equality

Extraneous Roots

Intercepts

Solutions

Set Notation

# 2.05 Use rational functions to model and solve problems; justify results.

A. Graph $f(x) = \frac{2}{x+5}$. Identify intercepts and any vertical and horizontal asymptotes. State the domain and range of the function.

B. Graph $f(x) = \frac{x-3}{x^2-2x-15}$. Identify intercepts and any vertical and horizontal asymptotes. State the domain and range of the function.

C. Graph $f(x) = \frac{x^3+x^2}{x^2+2x-8}$. Identify intercepts and any vertical and horizontal asymptotes. State the domain and range of the function.

D. Explain how $f(x) = \frac{x^2-9}{x-3}$ and $g(x) = x + 3$ are similar and different. Include the graphs and a comparison of the functions' asymptotes, intercepts, and domain.

E. Consider $f(x) = \frac{\mathbf{a}}{x+\mathbf{b}}$. As the value of **a** increases/decreases, how does the graph of $f(x)$ change? As **b** increases/decreases, how does the graph of $f(x)$ change?

F. Solve $\frac{x+6}{2x+6} = \frac{3x-2}{2x+1}$ algebraically; justify steps used. Identify the solution(s) graphically.

G. The function $d(x) = \frac{8710x^2 - 69400x + 470000}{1.08x^2 - 324x + 82200}$ can be used to accurately model the braking distance (feet) for cars traveling between 20 and 70 miles per hour. If a car doubles its speed, how does the stopping distance change? (double? triple?) If it takes a car 340 feet to stop safely, how fast was the car traveling?

H. Consider $g(x) = \mathbf{h} + \frac{1}{(x+\mathbf{k})^2}$. As **h** increases/decreases, how does the graph of $g(x)$ change? As **k** increases/decreases, how does the graph of $g(x)$ change?

***Vocabulary Concepts Skills***

I. When you eat sugary foods, the amount of acid in your mouth temporarily increases. This causes your mouth's pH level to decrease. The pH level $t$ minutes after eating can be approximated by the function $P(t) = \frac{65t^2 - 204t + 2340}{10t^2 + 360}$. Tooth decay can occur if the pH level in your mouth falls too low and remains low for a period of time. How long after sugary foods are eaten is the lowest pH level reached? What is the lowest pH level? As time passes, what value does the pH level approach?

J. A truck traveling at constant speed on a reasonably straight, level road burns fuel at the rate of $g(x)$ gallons per mile, where $x$ is the speed of the truck (miles per hour) and $g(x) = \frac{800 + x^2}{200x}$. If fuel costs $2.40 per gallon, find the cost function, $c(x)$, that expresses the amount of fuel needed for a 500-mile trip as a function of speed. What driving speeds will make the cost of the fuel for the trip less than $450? What driving speed will minimize the cost of fuel for the trip?

K. The function that describes gravitational acceleration ($\frac{\text{meters}}{\text{second}^2}$) of an object relative to the Earth is $g(r) = \frac{3.987 \cdot 10^{14}}{(6.378 \cdot 10^6 + r)^2}$ where $r$ is the distance in meters above the earth's surface. Use the graph of $g(r)$ to explain if it is possible to escape Earth's gravity.

L. The expected population of bears, $P(t)$, in a national park for the next 100 years ($t$) is modeled by $P(t) = \frac{500 + 250t}{10 + 0.5t}$. What is the initial bear population? Find the population after 10, 40, and 100 years. If the bear population continues to grow according to projections, what is the maximum expected population?

M. One simple plan for a state income tax requires those persons with incomes of $10,000 or less to pay no tax and those persons with incomes greater than $10,000 to pay a tax of 8% only on the amount over $10,000. Algebraically, what does this tax plan look like?
A person's effective tax rate is defined as the percent of total income that is paid in tax. Based on this definition and the plan outline, could any person's effective tax be 7%? Explain your answer. Include an example to justify your conclusion. Algebraically, what does this effective tax rate look like?
Based on this definition and the plan outline, could any person's effective tax be 8%? Explain your answer. Include an example to justify your conclusion.

***Vocabulary Concepts Skills***

Graph

Factor

Maximum

Minimum

Increasing

Decreasing

Domain

Range

Independent

Dependent

Transformations

Degree

Coefficients

Intercepts

Solutions

Zeros

Roots

$f(x) = ax^3 + bx^2 + cx + d$

# 2.06 Use cubic equations to model and solve problems.

A. If $f(x) = 2x^3 + 3x^2 + \mathbf{d}$, as **d** increases/decreases, how does the graph of $f(x)$ change?

B. Compare $f(x) = -3x^3 + x + 7$ and $g(x) = 3x^3 + x + 7$. Identify similarities and differences.

C. If $f(x) = \mathbf{a}x^3 - 6.5x^2 + x + 7$ and $\mathbf{a} \geq 1$, as **a** increases, how does the graph of $f(x)$ change?

D. Solve $3x^3 + 2x^2 = 9x + 6$ graphically.

E. The function $P(x) = 0.018x^3 - 0.687x^2 + 6.638x + 16$ describes the value of a precious metal over a 23-month period. During which month did the metal achieve its greatest value? Determine the lowest value since then. Describe the value of the metal over the last ten months. If the $P(x)$ continues to model the value of the precious metal, will the value exceed its previous greatest value in the next six months or will it drop below the previous low value (not the initial value)?

F. The function $M(x) = -0.287x^3 + 8.8x^2 - 59.843x + 220.7$ describes the incidence of measles (per 100,000) for the period 1940-1960 ($x = 0$ for 1940). In what year was the greatest incidence of measles reported? According to the definition of $M(x)$, what is the y-intercept? Identify periods of increasing /decreasing frequency of the disease. If the function continues to model the disease beyond 1960, when did the incidence of measles approximate zero? What are some variables that may have affected the incidence of measles over the period 1940-1960?

G. An open box is to be created from a nine-inch by twelve-inch piece of posterboard by cutting congruent squares from each corner and folding up the sides. Detemine the size of the square that should be cut from each corner to produce the box with maximum volume.

***Vocabulary Concepts Skills***

Graph

Factor

Maximum

Minimum

Increasing

Decreasing

Domain

Range

Independent

Dependent

Transformations

Radical

Index

Radicand

Coefficients

Intercepts

Properties of Equality

Set Notation

Solutions

Zeros

$f(x)=\sqrt{\mathrm{a}x+\mathrm{b}}+\mathrm{c}$

# 2.07 Use equations with radical expressions to model and solve problems; justify results.

A. In a hurricane, the mean sustained wind velocity *V*, measured in meters per second, is given by $V=6.3\sqrt{1013-P}$ where *P* is the air pressure, measured in millibars (mb), at the center of the hurricane. What happens to wind velocity in a hurricane when air pressure decreases?

B. Solve $\sqrt{x^2+5x+4}=2-x$ exactly for ***x***. Justify each step.

C. Solve $\sqrt{6x+4}=x+1$ exactly for ***x***. Justify each step.

D. Solve $y+5=\sqrt[3]{2y-3}$ for ***y***. Justify each step.

E. The per barrel price (in dollars) of petroleum for a recent 20-month period is described by $p(m)=\sqrt{7m^2+8m+377}$. Identify the minimum and maximum prices for the period. At what point did the price of petroleum first exceed $35? If the model continues to be accurate, will the price per barrel reach $60? When?

F. The price of a computer over a 25-month period is described by $f(x)=1050-\sqrt[3]{9x^5+5x+503}$. Identify the minimum and maximum prices for the period. At what point did the price of the computer first reach $800? If the model continues to be accurate, when will the price reach half of its initial price?

G. If $f(x)=\sqrt{\mathbf{a}x+9}$ and $\mathbf{a}>\mathbf{0}$, as **a** increases, how does the graph of *f*(*x*) change?

H. If $f(x)=\sqrt{3x+\mathbf{b}}$, as **b** increases, how does the graph of *f*(*x*) change?

I. For $f(x)=\sqrt{x+3}+\mathbf{c}$, as **c** increases/decreases, how does the graph of *f*(*x*) change?

***Vocabulary Concepts Skills***

Graph

Factor

Maximum

Minimum

Increasing

Decreasing

Domain

Range

Independent

Dependent

Transformations

Coefficients

Intercepts

Properties of Equality

Set Notation

Number Line

Solutions

Zeros

Conjunction

Disjunction

$f(x) = |ax + b|$

# 2.08 Use equations and inequalities with absolute value to model and solve problems; justify results.

**A.** A fast-moving cold front in the Northeast can cause temperatures to drop very quickly then rise again. The following data uses $t$ as the hours since midnight on a day the cold front moves in, and $T$ as the temperature in degrees Fahrenheit.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $T$ | 3 | 1 | –1 | –3 | –5 | –7 | –5 | –3 | –1 | 1 | 3 |

The absolute value function has a similar shape. Use a transformation of $f(x) = |x|$ to model this data.

B. Consider the graph of $y = |\mathbf{a}x + 5|$ when $\mathbf{a} \geq 2$. As **a** increases, how does the graph change? What happens to the vertex of $y = |\mathbf{a}x + 5|$?

C. Consider the graph of $y = |2x + \mathbf{b}|$ when $\mathbf{b} \geq 0$. As **b** increases, how does the graph change?

D. What plane figure is described by $y \geq |2x - 3|$ and $y \leq 6 - |2x - 3|$?

E. Given $y \geq |2x - 3|$, find another equation of the form $y \leq |ax + b|$ so that a system of inequalities exists that describes a kite with one vertex at (0, 3).

F. Solve $|3x - 4| \leq 17$ for $x$. Justify each step.

G. Solve $|16 – 3x| = |x| + 3$ for $x$.

*Vocabulary*
*Concepts*
*Skills*

Conics

Radius

Center

Vertex

Axis of Symmetry

Completing the Square

$y - k = a(x - h)^2$

$(x - h)^2 + (y - k)^2 = r^2$

$x^2 + y^2 + Dx + Ey + F = 0$

# 2.09 Use the equations of parabolas and circles to model and solve problems; justify results.

A. In the coordinate plane, a line parallel to the $x$-axis intersects the $y$-axis at (0, 4). This line also intersects a circle in two points. The circle has a radius of 10 and its center is at the origin. What are the coordinates of the points of intersection? What are the coordinates of the points of intersection if the circle's center is at (1, 2)? (-3, -4)?

B. Write the equations for all circles with a radius of 2.5 and tangent to lines $x = -1.5$ and $y = 3$.

C. Identify exactly the vertex of $y = x^2 - 7x + 9$. Identify exactly the $x$- and $y$-intercepts.

D. Identify exactly the vertex of $x = 3y^2 - 12y + 2$. Identify exactly the $x$- and $y$-intercepts.

E. Graph $x^2 + y^2 = 8y - 6y - 3$. Identify exactly the $x$- and $y$-intercepts.

***Vocabulary Concepts Skills***

Graph

Intersection

Substitution

Matrix Equation

Linear

Quadratic

Cubic

Exponential

Rational

Radical

Absolute Value

Circle

# 2.10 Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.

A. For the period 1970-1998 ($x = 0$ for 1970), $f(x) = 0.71x^2 + 2.15x + 67.53$ models US exports and $g(x) = 0.82x^2 + 6.42x + 55.07$ models US imports. Find the years when US trade was balanced, $f(x) = g(x)$. Identify graphically and define algebraically the US trade surplus/deficit, according to the functions provided.

B. A luxury car's value is represented by the equation $y_1 = 50(0.822)^x$. A sports utility vehicle's (SUV) value is represented by the equation $y_2 = 30(0.884)^x$. In both functions, $\boldsymbol{y}$ is the value ($1000's) of the automobile after $\boldsymbol{x}$ years. Assuming one of each model is purchased on the same day, how long before the luxury car is only worth $1000 more than the SUV? How long until the two automobiles are equal in value? How long before the SUV is worth $1000 more than the luxury car?

C. Write the system of inequalities that describes a triangular region with one vertex at (7, 5) and another on the $x$-axis. None of the sides can be horizontal or vertical.

D. Find the solution for the system:

$$6x - 3y + 19z = 23$$
$$-5x + 11y - 5z = -14$$
$$x + 7y + 4z = 6$$

E. Find the solution for the system:

$$-w + 2.4x + 4.5y - 7.1z = 19$$
$$3w + 7.7x - 5.8y + 2z = 0$$
$$1.3w - 4.3x + 9.2y - z = 23.4$$
$$-5.3w + 6.6x + 1.6y + 7.9z = 4.4$$

F. Write a system of inequalities that defines a parallelogram in the second quadrant with one vertex at (-3, 4). The sides cannot be vertical or horizontal.